

Inverse Trigonometric Functions

Part - 2



ASSERTION AND REASON BASED MCQs

(1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

- (A) Both A and R are true and R is the correct explanation of A
- (B) Both A and R are true but R is NOT the correct explanation of A
- (C) A is true but R is false
- (D) A is false and R is True

Q. 1. Assertion (A): $\sin^{-1}\left(\sin \frac{2\pi}{3}\right) = \frac{2\pi}{3}$

Reason (R): $\sin^{-1}(\sin \theta) = \theta$, if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Ans. Option (D) is correct.

Explanation:

The principal value branch of $\sin^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Let $x = \sin \theta \Rightarrow \theta = \sin^{-1}x$

$\sin^{-1}(\sin \theta) = \sin^{-1}x = \theta$

$\sin^{-1}(\sin \theta) = \theta$, if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Hence R is true.

$\sin^{-1}\left(\sin \frac{2\pi}{3}\right) \neq \frac{2\pi}{3}$, since $\frac{2\pi}{3} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Hence A is false.

Q. 2. Assertion (A): Range of $\tan^{-1}x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Reason (R): Domain of $\tan^{-1}x$ is $\mathbb{R}$.

Ans. Option (B) is correct.

Explanation: Domain of $\tan x$ is the set $\{x : x \in \mathbb{R} \text{ and } x \neq (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}\}$ and Range is $\mathbb{R}$.

$\Rightarrow \tan x$ is not defined for odd multiples of $\frac{\pi}{2}$.

If we restrict the domain of tangent function to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then it is one-one and onto with its range as $\mathbb{R}$. Actually $\tan x$ restricted to any of the intervals $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right), \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ etc., is bijective and its range is $\mathbb{R}$.

Thus $\tan^{-1}x$ can be defined as a function whose domain is $\mathbb{R}$ and range could be any of the intervals $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right), \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ and soon.

$\therefore$ Both A and R are true but R is not correct explanation of A.

Q. 3. Assertion (A): Principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is $\frac{\pi}{4}$

Reason (R): Principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ is $\frac{\pi}{3}$

Ans. Option (C) is correct.

Explanation:

$$\begin{aligned} \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) &= \sin^{-1}\left(\sin \frac{\pi}{4}\right) \\ &= \frac{\pi}{4} \end{aligned}$$

$$\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = y$$

$$\cot y = \frac{-1}{\sqrt{3}}$$

$$= -\cot\left(\frac{\pi}{3}\right)$$

$$= \cot\left(\pi - \frac{\pi}{3}\right)$$

$$= \cot\left(\frac{2\pi}{3}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{2\pi}{3}$$

Hence Assertion is correct and Reason is incorrect.

Q. 4. Assertion (A): Range of $\cot^{-1}x$ is $(0, \pi)$

Reason (R): Domain of $\tan^{-1}x$ is $\mathbb{R}$.

Ans. Option (B) is correct.

Q. 5. Assertion (A): Principal value of $\cos^{-1}(1)$ is π

Reason (R): Value of $\cos 0^\circ$ is 1

Ans. Option (D) is correct.

Explanation: In case of Assertion:

$$\cos^{-1}(1) = y$$

$$\cos y = 1$$

$$\cos y = \cos 0^\circ \quad [\because \cos 0^\circ = 1]$$

$$\therefore y = 0$$

$\Rightarrow$ Principal value of $\cos^{-1}(1)$ is 0

Hence Assertion is incorrect.

Reason is correct.

